Differential equation with economic interpretation

If C is the total cost corresponding to a quantity produced x, it is known that the marginal cost is equal to the average cost. Show that the total cost is directly proportional to the quantity produced.

Solution

The marginal cost is defined as: $\frac{dC}{dx}$ and the average cost: $C_m = C/x$. So, having $C' = C/x$, we need to show that $C = kx$.

$$\frac{dC}{dx} = \frac{C}{x}$$
$$\frac{1}{C}dC = \frac{1}{x}dx$$
$$ln(C) = ln(x) + k$$
$$C = e^{ln(x)+k}$$
$$C = xK$$

In this way, we corroborate that C is directly proportional to the quantity x. We cannot know exactly the proportion as we do not have the value of w. For example, if $w = 2$, then the proportion is 2 to 1. A doubling of the quantity quadruples the cost.